## CS/Math 320 Discrete Mathematics

*Fall 2017*

## Test 2 5:30PM - 6:45PM

*Each problem is worth 15 points.*

Problem 1. What is the cardinality of {a, {a}, {a, {a}}}?

Answer: 3

Problem 2. Let A be a set. Show that $\varnothing \times A = A \times \varnothing = \varnothing$.

Answer:

$\varnothing \times A = \{(x,y) \mid x \in \varnothing \wedge y \in A\} = \{(x,y) \mid F \wedge y \in A\} = \{(x,y) \mid F\} = \varnothing$

$A \times \varnothing = \{(x,y) \mid x \in A \wedge y \in \varnothing\} = \{(x,y) \mid y \in A \wedge F\} = \{(x,y) \mid F\} = \varnothing$

Problem 3. Let A, B, and C be sets. Show that $(B - A) \cup (C - A) = (B \cup C) - A$

Answer:

$(B - A) \cup (C - A)$

$= \{x \mid x \in B\text{-}A \vee x \in (C - A)\}$

$= \{x \mid x \in B \wedge x \notin A \vee x \in C \wedge x \notin A\}$

$= \{x \mid (x \in B \vee x \in C) \wedge x \notin A\}$

$= \{x \mid x \in B \cup C \wedge x \notin A\}$

$= (B \cup C) - A$

Problem 4. Suppose that the universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Using computer representation:

- **a)** Find the set corresponding to bit string 01 0111 1000
- **b)** Express set {2,3,4,7,8,9} as a bit string

Answer:

- a) {2, 4, 5, 6, 7}
- b) 0111001110

Problem 5. Give an explicit formula for a function from the set of integers to the set of positive integers that is

- a) one-to-one, but not onto.
- b) onto, but not one-to-one.
- c) one-to-one and onto.
- d) neither one-to-one nor onto.

Answer: There are many solutions, for example the below:

a) f: Z→Z+ such that f(n) = 3(-n) if n<0 or 3n+1 if n>=0
b) f: Z→Z+ such that f(n) = |n|+1
c) f: Z→Z+ such that f(n) = 2n if n>0 or 2(-n)+1 otherwise
d) f: Z→Z+ such that f(n) = $1+2n^2$

Problem 6. Let f be a function from R to R and $f(x)=x^2$. Find $f^{-1}(\{x \mid 0 < x < 1\})$.

Answer:
$f^{-1}(\{x \mid 0 < x < 1\}) = \{x \mid 0<f(x)<1\} = \{x \mid 0<x^2<1\} = \{x \mid -1 < x < 1 \wedge x \neq 0\}$

Problem 7.
Let $f$ be a function from $A$ to $B$. Let $S$ be a subset of $B$.
Show that $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$.

Answer:
$f^{-1}(\text{complement}(S)) = \{x \mid f(x) \notin S\} = \{x \mid x \notin f^{-1}(S)\} = \text{complement}(f^{-1}(S))$