Assignment #2

Posted on Feb. 26 – due by March 5 at 4:00pm

Please hand in a hard copy of your homework (handwritten is OK) at the start of the class or email a digital version to our TA, Thuy Do (Thuy.Do001@umb.edu).

Question 1:

For each of the following functions, show that it is primitive recursive. Use a “relaxed” mathematical argument such as we did starting on slide 7 on February 26. You do not have to provide a precise proof as we did on the earlier slides, but you can if you like. In your arguments, you can use any functions that we have already shown in class to be primitive recursive without having to prove it again.

(a) \( \gcd(x_1, x_2) \) (greatest common divisor; let us define that \( \gcd(0, 0) = 0 \))

(b) \( f(x) = \lfloor \sqrt{x} \rfloor \)

(c) \( g(x_1, x_2) = \min(x_1^2, x_2, x_1 x_2^2) \) (Here, \( \min \) returns the smaller of its two inputs).

(d) \( k(x_1, x_2) \) is the number of twin primes whose smaller twin is greater than \( x_1 \) and less than \( x_2 \). Twin primes are pairs of primes whose difference is two. For example, (3, 5) and (11, 13) are twin primes.
Question 2:

Write an \( L \) program to compute each of the following functions as they are defined on the slides. You can use macros but have to provide code for any functions or predicates used as macros. You can use the Haskell tools to test your code. You can write down the code in standard \( L \) notation or in the Haskell (.l) format.

(a) \( Q(x, y) = x \mid y \)

(b) \text{Prime}(x)

(c) \( p_n \) \text{ (bonus question)}