Question 1: A Variation of the Language $L$

Let us consider a new language $L'$. This language is identical to $L$, except that instead of the instruction types

\[
V \leftarrow V \\
V \leftarrow V + 1 \\
V \leftarrow V - 1 \\
\text{IF } V \neq 0 \text{ GOTO L}
\]

we now have the instruction types

\[
V \leftarrow V \\
V \leftarrow V + 3 \\
V \leftarrow V - 2 \quad \text{(again, if } V \text{ is 0 or 1, it becomes 0 after this operation)} \\
\text{IF } V = 1 \text{ GOTO L}
\]

(a) Show that the language $L'$ is at least as powerful as $L$, that is, can compute all functions that $L$ can compute. **Hint:** You need to show for each instruction type in $L$ how it can be simulated in $L'$, i.e., how we could translate it into $L'$ code that has the same effect.

(b) Write down the entire $L'$ code for the new universal programs $U'_n$ that can execute the code of any $L'$ program. Of course you can use macros, and you can reuse most of the code that we wrote for programming $U_n$ (see slides and textbook).
**Question 2: Some Set Operations**

Let A and B be sets. Prove or disprove:

(a) For all sets A and B, if A is recursive and B is r.e., then $A \cup B$ is also r.e.

(b) For all sets A and B, if A and B are both recursive, then $A - B$ is also recursive.

(c) If $A \subseteq B$ and A is r.e., then B is r.e.

(d) If $A \cup B$ is r.e., then both A and B are r.e.

(e) If A is r.e, then $\neg A$ is also r.e.

**Question 3 (Bonus Question): What about These Sets?**

For each of the following sets, determine whether it is recursive, r.e., or neither. Prove your answer. Of course, if you prove that a set is recursive, it is clear that it is also r.e., and you do not have to prove that.

(a) $A = \{ x \in \mathbb{N} \mid x^2 > 1000 \}$

(b) $B = \{ x \in \mathbb{N} \mid x$ is the number of a program that computes the function $f(n) = 0 \}$

(c) $C = \{ x \in \mathbb{N} \mid x$ is the number of a program that terminates on input 12 after 50 or more steps\}$

(d) $D = \{ x \in \mathbb{N} \mid x$ is the number of a program whose output is defined for at least 3 different inputs\}$