Question 1: Playing around with Strings

Show every step of your calculations for the following questions. Please always complete your calculations; for example, in your answer write 3125 instead of 55.

(a) Which integer is associated with the string “xmas” on the alphabet $A = \{a, s, m, n, x\}$?

(b) What is the string on the alphabet $B = \{b, m, n, u\}$ that is associated with the integer 73?

(c) Let $n$ be a natural number and $w_A$ and $w_B$ be the strings associated with $n$ on the alphabets $A$ and $B$, respectively, where $|A| = 2$ and $|B| = 3$. Give an example of a number $n$ for which the length of $w_A$ is 4 and the length of $w_B$ is 3.

(d) Compute $\text{UPCHANGE}_{2,5}(95)$.

(e) Compute $\text{DOWNCHANGE}_{3,8}(199)$.

(f) Compute $\text{LTEND}_{6}(100)$.

(g) Given the alphabet $C = \{a, b\}$, what is the smallest number whose associated string on $C$ has a length of four?
Question 2: Some Coding with the $\mathcal{L}_1$ and $\mathcal{L}_2$ Languages

a) Write an $\mathcal{L}_1$ program that computes a function $f$ that multiplies its input (when interpreted as number) by 3. For example:

\[
\begin{align*}
    f(s_1) &= s_1 s_1 s_1 \\
    f(s_1 s_1) &= s_1 s_1 s_1 s_1 s_1 s_1 \\
    f(0) &= 0 \quad \text{(remember: 0 stands for the empty string)}
\end{align*}
\]

b) Write an $\mathcal{L}_2$ program that computes a function $g$ that sorts the symbols in its input string in ascending order. For example:

\[
\begin{align*}
    g(s_2 s_1 s_2) &= s_1 s_2 s_2 \\
    g(s_2 s_1 s_2 s_2 s_1 s_2 s_2) &= s_1 s_1 s_1 s_2 s_2 s_2 s_2 \\
    g(0) &= 0
\end{align*}
\]