Question 1: Playing around with Strings

Show every step of your calculations for the following questions. Please always complete your calculations; for example, in your answer write 3125 instead of $5^5$.

(a) Which integer is associated with the string “zork” on the alphabet $A = \{a, o, r, k, z\}$?

(b) What is the string on the alphabet $B = \{a, b, c\}$ that is associated with the integer 134?

(c) Compute $\text{UPCHANGE}_{3,5}(100)$.

(d) Compute $\text{DOWNCHANGE}_{7,9}(201)$.

(e) Compute $\text{LTEND}_{4}(37)$.

(f) Given the alphabet $C = \{x, y, z\}$, what is the least integer whose associated string on $C$ has a length of four?
Question 2: Ordering with $\mathcal{L}_2$

Write an $\mathcal{L}_2$ program (you can use macros) that computes the following function $f$:

$$f(x, y) = \begin{cases} s_1, & \text{if } x < y \\ s_2, & \text{otherwise} \end{cases}$$

Here, $x < y$ means that the string $x$ precedes the string $y$ in alphabetical order. For example, it is true that $s_1s_2s_2s_1 < s_2s_1$, $s_1s_1s_1 < s_1s_1s_2$, and $s_2s_2 < s_2s_2s_2$.

Question 3 (Bonus Question): Ordering with Post-Turing

a) Write a Post-Turing program (you can use macros) using the alphabet $A = \{s_1, s_2\}$ that computes the same function $f$ from Question 2.

b) Write down the list of successive tape configurations that your program generates during the computation of $f(s_1s_2s_1, s_1s_2s_2s_1)$. 