You proved in a previous homework assignment that CLIQUE is NP-complete, essentially by showing that

\[ \text{VERTEX COVER} \leq_{P} \text{INDEPENDENT SET} \leq_{P} \text{CLIQUE} \]

Here’s another way to show that CLIQUE is NP-complete, using the fact that 3-SAT is NP-complete. First, we note that CLIQUE is checkable in polynomial time. This is almost trivial, and you don’t have to prove it.

So what remains is to show that 3-SAT \( \leq_{P} \) CLIQUE. Here’s how we’ll do it: Suppose that we have an instance of the 3-SAT problem. This is just a 3-SAT expression involving the \( n \) Boolean variables \( v_1, v_2, \ldots, v_n \)

The 3-SAT expression must be of the form

\[ c_1 \land c_2 \land \cdots \land c_k \]

where each \( c_j \) is a clause of the form

\[ c_j = \gamma_1^{(j)} \lor \gamma_2^{(j)} \lor \gamma_3^{(j)} \]

where each \( \gamma_i^{(j)} \) is a literal—that is it is either one of the variables \( v_m \) or its negation \( \overline{v_m} \).

Starting with this expression, we need to construct an instance of CLIQUE. Here’s how we do it:

1. We construct a graph \( G = (V, E) \) as follows: For each clause

   \[ c_j = \gamma_1^{(j)} \lor \gamma_2^{(j)} \lor \gamma_3^{(j)} \]

   we create a group of three vertices. We label each vertex with either the variable or its negation that the literal in the clause indicates. For instance, if \( \gamma_1^{(j)} = v_3 \), then the vertex corresponding to \( \gamma_1^{(j)} \) will be labelled with \( v_3 \). And if on the other hand \( \gamma_1^{(j)} = \overline{v_3} \), then the vertex corresponding to \( \gamma_1^{(j)} \) will be labelled with \( \overline{v_3} \). In either case, this vertex will be in “group \( j \)”, because it comes from the \( j \)th clause in the 3-SAT expression.

   When we have done this for all the clauses, we will have a total of \( 3k \) vertexes, in groups of three.
2. Now we have created all the vertices $V$ in our graph $G$, and we have to create the set of edges. We create an edge from one vertex to another provided
   
   - The two vertexes are not in the same group, and
   - The two vertexes are *consistent*, in the sense that neither one holds the negation of the other. (So $v_3$ and $\overline{v_7}$ would be consistent, and $v_3$ and $v_3$ would also be consistent, but $v_3$ and $\overline{v_3}$ would not be consistent.)

3. Your job is to prove that this graph has a clique of size $k$ if and only if the original 3-SAT expression is satisfiable. (And remember where $k$ comes from!)

4. Once you have done this, you are almost finished. You still have to point out one other thing—and no, it’s *not* that CLIQUE is in NP—I already told you that you could assume that. What else do you have to point out? It should be very simple.